

B.Sc. II:-

Unit: (II) [Interference]

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पृष्ठ 92

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[Physical Optics]

पूर्वाञ्चल विश्वविद्यालय, जौनपुर

कक्षा -----	विषय -----
अनुक्रमांक -----	प्रश्न-पत्र का नाम तथा कोड नं० -----
नामांकन संख्या -----	तिथि -----

कक्ष परिप्रेक्षक के हस्ताक्षर

परीक्षार्थी इस पृष्ठ से लिखना प्रारम्भ करें तथा उत्तर-पुस्तिका के दोनों ओर लिखें।

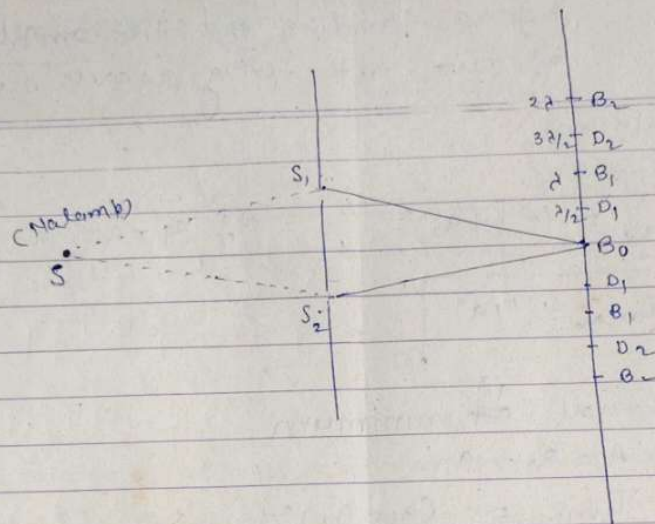
Conditions for interference:-

Source should be monochromatic i.e. it should emit waves of single wave length or single colour. Sodium lamp is the best monochromatic source.

2) Two source should be coherent i.e. it emit light waves of the same frequency and are always in same phase with each other. This is possible only when both source are

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When path difference between two waves

$$\Delta x = 0, \lambda, 2\lambda, 3\lambda, \dots = n\lambda$$

or phase difference

$$\Delta \phi = 0, 2\pi, 4\pi, 6\pi, 8\pi, \dots = 2n\pi$$

} maxima

(ii) Similarly, minimas (dark fringes) are formed at those points where crest of one wave meets with trough of another wave. This is possible, when path difference

$$\Delta x = \lambda/2, 3\lambda/2, 5\lambda/2, 7\lambda/2, \dots = (2n+1)\lambda/2$$

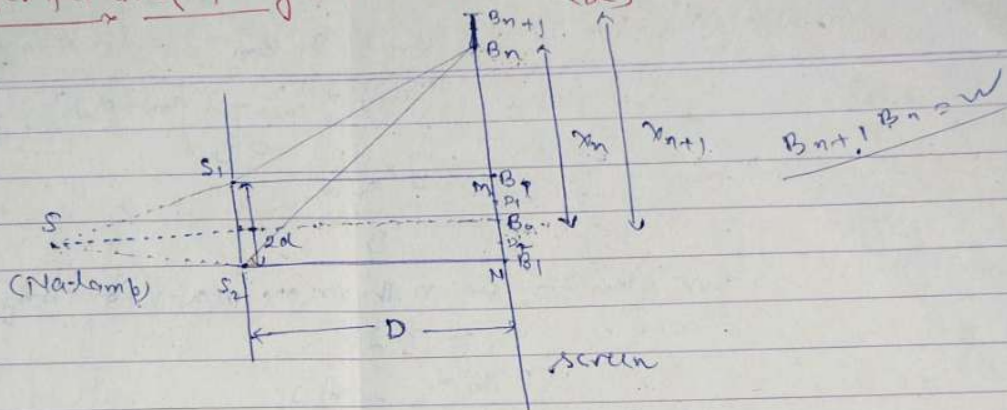
$$n = 0, 1, 2, 3, \dots$$

or phase difference

$$\Delta \phi = \pi, 3\pi, 5\pi, 7\pi, \dots = (2n+1)\pi$$

$$n = 0, 1, 2, 3, \dots \text{ minima}$$

Theory of interference fringes :- $[W = \frac{D\lambda}{2d}]^2$



$B_0 B_n = x_n$ (distance of n -th max from central max)

$S_1 \rightarrow D S_1 M B_n$

$$S_1 B_n^2 = D^2 + (x_n - d)^2$$

$$\therefore S_1 B_n = [D^2 + (x_n - d)^2]^{1/2}$$

$$= (D^2)^{1/2} \left[1 + \frac{(x_n - d)^2}{D^2} \right]^{1/2}$$

$$S_1 B_n = D + \frac{1}{2} \frac{(x_n - d)^2}{D} \quad S_1 B_n = D \left[1 + \frac{1}{2} \frac{(x_n - d)^2}{D^2} \right] \text{ (using Binomial theorem)}$$

In $\Delta S_2 N B_n \rightarrow S_2 B_n^2 = D^2 + (x_n + d)^2$

$$S_2 B_n = [D^2 + (x_n + d)^2]^{1/2}$$

$$= (D^2)^{1/2} \left[1 + \frac{(x_n + d)^2}{D^2} \right]^{1/2}$$

$$S_2 B_n = D \left[1 + \frac{1}{2} \frac{(x_n + d)^2}{D^2} \right]$$

$$= D + \frac{(x_n + d)^2}{2D}$$

Intensity of Max :- If a_1 and a_2 are the amplitude of two interfering waves, then

amplitude is maximum

$$A = a_1 + a_2$$

$$I_{\max} = (a_1 + a_2)^2 \quad \text{if } a_1 = a_2 = a$$

$$\therefore I_{\max} = 4a^2$$

Similarly amplitude ^{is} minimum

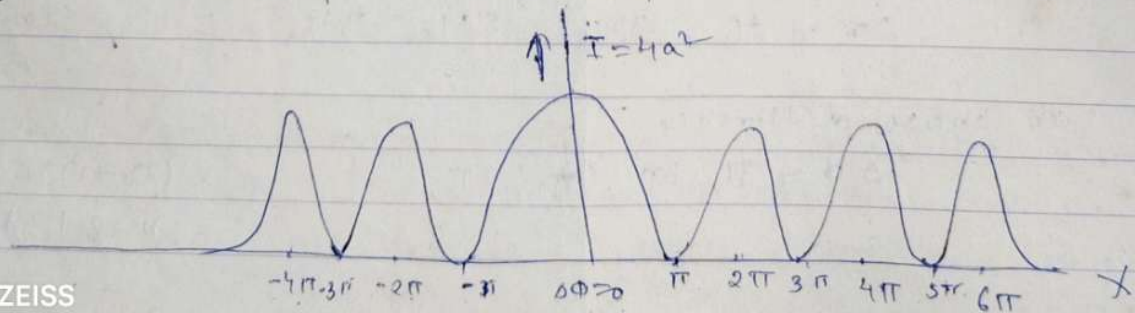
$$A = a_1 - a_2$$

$$I_{\min} = (a_1 - a_2)^2$$

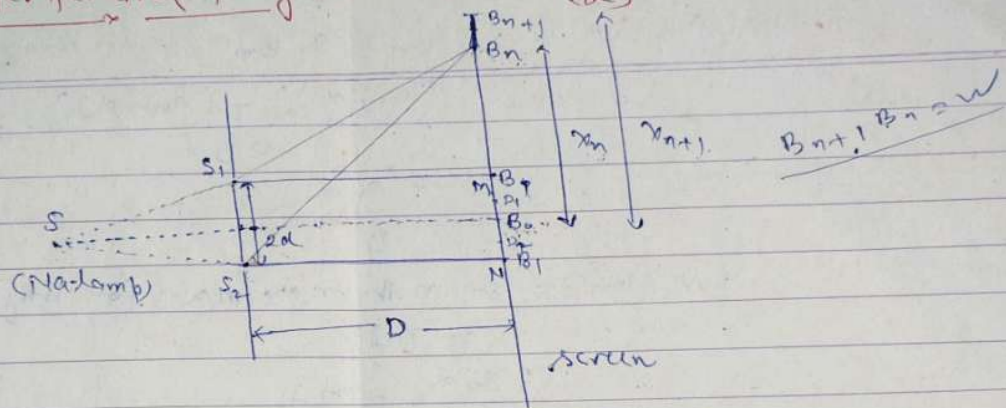
$$= 0$$

$$\text{Average intensity} = \frac{I_{\max} + I_{\min}}{2} = \frac{4a^2 + 0}{2} = 2a^2$$

Thus we see that in case of interference, average intensity remains ^{constant}. The source as would be if there were no interference. $\rightarrow$ This law of conservation of energy is obeyed in case of interference.



Theory of interference fringes :- $[w = \frac{D\lambda}{2d}]$



$B_0B_n = x_n$ (distance of n th max from central max) .

In ΔS_1MB_n

$$S_1B_n^2 = D^2 + (x_n - d)^2$$

$$\therefore S_1B_n = [D^2 + (x_n - d)^2]^{1/2}$$

$$= (D^2)^{1/2} \left[1 + \frac{(x_n - d)^2}{D^2} \right]^{1/2}$$

$$S_1B_n = D + \frac{1}{2} \frac{(x_n - d)^2}{D} \quad S_1B_n = D \left[1 + \frac{1}{2} \frac{(x_n - d)^2}{D^2} \right] \text{ (using Binomial theorem)}$$

$$\text{In } \Delta S_2NB_n \rightarrow S_2B_n^2 = D^2 + (x_n + d)^2$$

$$S_2B_n = [D^2 + (x_n + d)^2]^{1/2}$$

$$= (D^2)^{1/2} \left[1 + \frac{(x_n + d)^2}{D^2} \right]^{1/2}$$

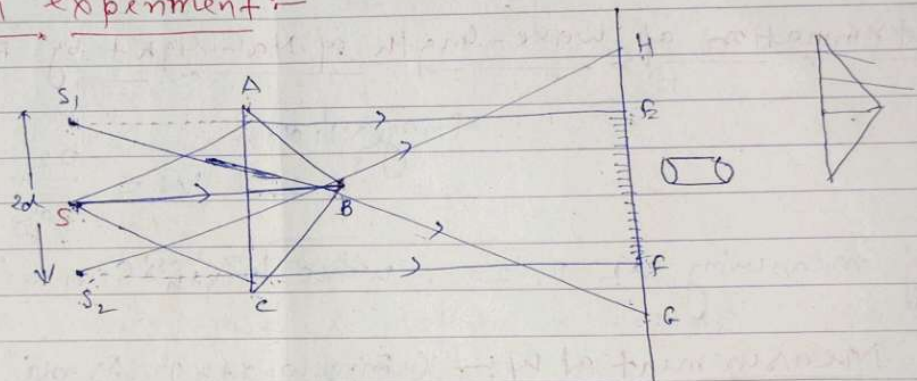
$$S_2B_n = D \left[1 + \frac{1}{2} \frac{(x_n + d)^2}{D^2} \right]$$

$$= D + \frac{(x_n + d)^2}{2D}$$

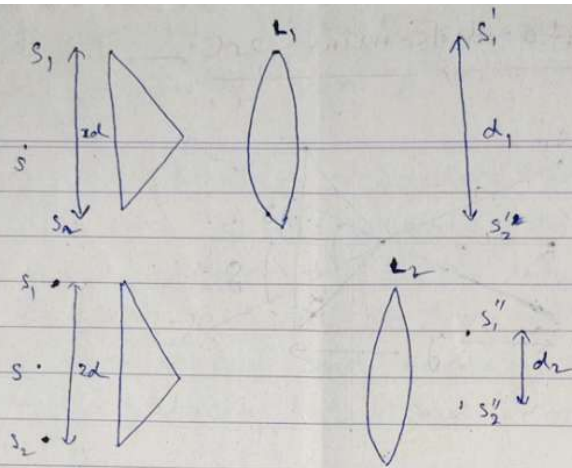
by same common source. (Fresnel's biprism exp.)

- (i) By division of wave front (Fresnel's biprism exp.)
- (ii) By division of amplitude (N.R. exp., Michelson interference)

Fresnel's biprism experiment:-



Fresnel's biprism consists of two prisms of very small angle $\frac{1}{2}^\circ$ placed base to base. S is slit illuminated by a monochromatic source (Na lamp). AE and BG are emergent rays in upper prism. These rays are divergent, so which produced back from virtual image at S_1 in upper prism. Similarly CF and BH are two emergent rays in lower prism, which form virtual image ~~at~~ at S_2 in lower prism. Since S_1 and S_2 are derived from same common sources (S), therefore S_1 and S_2 acts as two ~~co~~ coherent sources. Rays between E and G are supposed coming from S_1 . Similarly rays between F and H are supposed coming from S_2 . Thus coming from two $\rightarrow$



$$2d = \sqrt{d_1 d_2}$$

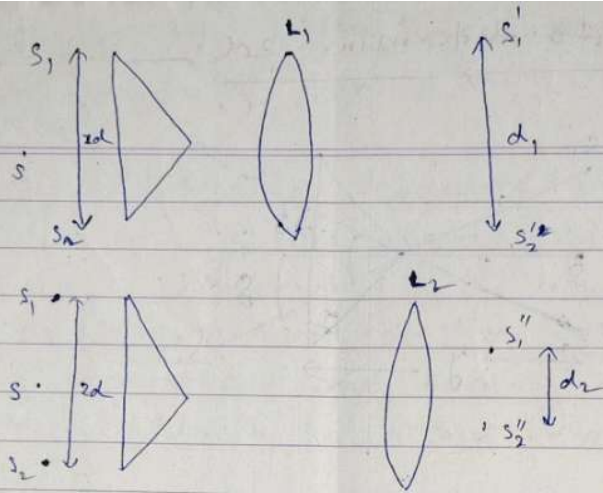
As shown above a convex lens is placed in position L_1 so that image S_1' and S_2' are screen in eye piece. distance S_1' and S_2' is measured by cross wire, let it be d_1 . Now lens is placed in position L_2 . S_1'' and S_2'' are the image of S_1 and S_2 in eye piece.

again distance between S_1' and S_2'' is measured with cross wire, let it be d_2 .

Then $2d$ is obtained by

$$2d = \sqrt{d_1 d_2}$$

putting value of w , $2d$, D in $(W = D^2 / 2d)$
 λ can be calculated.



$$2d = \sqrt{d_1 d_2}$$

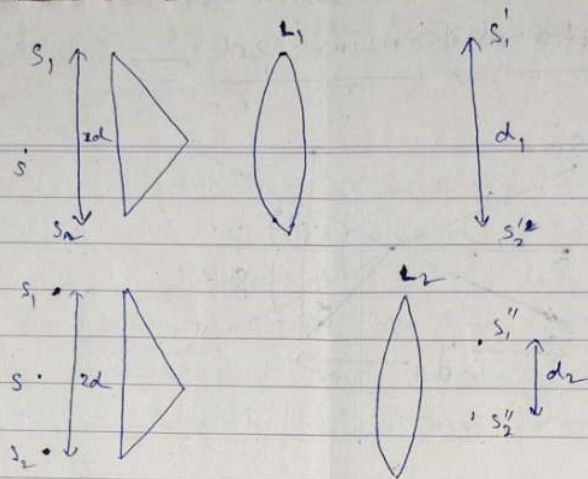
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Then $2d$ is obtained by

$$2d = \sqrt{d_1 d_2}$$

putting value of w , $2d$, D in $(w = D\lambda/2d)$
 λ can be calculated.



$$2d = \sqrt{d_1 d_2}$$

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again distance between S_1'' and S_2'' is measured with cross wire. let it be d_2 .

Then $2d$ is obtained by

$$2d = \sqrt{d_1 d_2}$$

putting value of w , $2d$, D in $(W = D^2 / 2d)$

λ can be calculated.

Suppose A and B are two virtual coherent sources. The point C is equidistance from A and B. When a transparent

plate G of thickness t and refractive index μ is introduced in the path of one of the beams. Since fringe which was originally at C shifts to P. The time taken by the wave from B to P in air is same as time taken by the wave from A to P partly through air and partly through the plate. If c_0 and c is velocity in air and medium then

$$\frac{BP}{c_0} = \frac{AP - x}{c_0} + \frac{x}{c}$$

Multiplying on both side by c_0

$$BP = AP - x + \frac{c_0}{c} x$$

$$\text{Since } \frac{c_0}{c} = \mu$$

$$BP - AP = \mu x - x$$

$$= (\mu - 1)x \quad \text{--- (I)}$$

If P is the point occupied by n th fringe then path difference

$$BP - AP = n\lambda$$

$$\therefore (\mu - 1)x = n\lambda \quad \text{--- (II)}$$

Also the distance x through which fringe is shifted

$$x_n = \frac{n\lambda D}{\alpha}$$

$$\text{where } \frac{\partial D}{\partial \alpha} = \beta = w \text{ (Fringe width)}$$

$$n=1$$

$$\lambda = \frac{n\lambda D}{d}$$

$$\therefore n\lambda = \frac{x d}{D}$$

$$\boxed{\therefore (\mu - 1)t = \frac{x d}{D}}$$

Therefore knowing x , distance through which central fringe is shifted, D , d and μ , thickness of the transparent plate can be calculated.

→ Interference of light :-
(Division of amplitude)

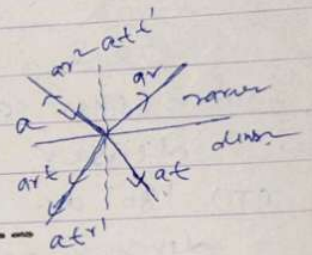
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कक्षा

विषय

अनुक्रमांक

प्रश्न-पत्र का नाम तथा कोड नं०

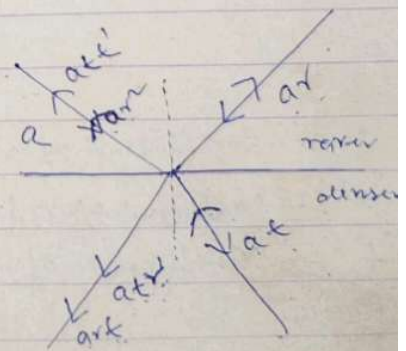
नामांकन संख्या

तिथि

कक्ष परिप्रेक्षक के हस्ताक्षर

परीक्षार्थी इस पृष्ठ से लिखना प्रारम्भ करें तथा उत्तर-पुस्तिका के दोनों ओर लिखें।

Stoke's treatment :- r = Coefficient of reflection
for rarer medium
 t = Coefficient of refraction
from rarer to denser
medium



r' = Coefficient of reflection in denser

t' = Coefficient of refraction from denser to rarer

$$a \times r' + a \times t' = 0$$

$$a r^2 + a t t' = a$$

$$\boxed{r' = -r}$$

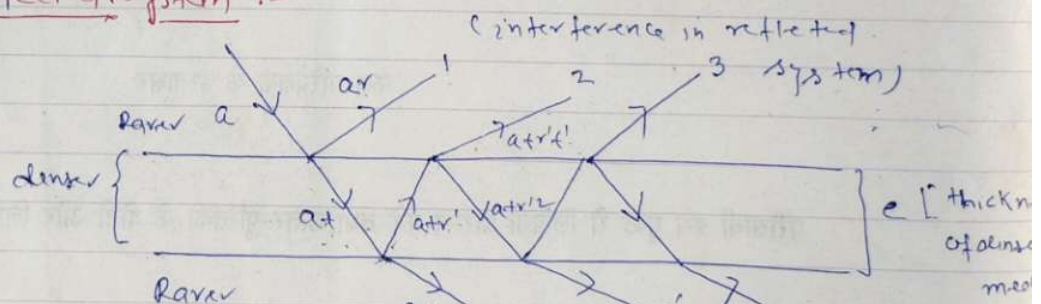
$t t' = 1 - r^2$ } Stoke's formula

$$\frac{\text{reflection}}{a} = r$$

$$\boxed{r = -r'}$$

When ever a ray is reflected in rarer medium backed by denser medium, there is a sudden phase change (π) 180° or path change $\lambda/2$. But a ray is reflected in denser medium, there is no any change in phase or path

Interference in reflected system :-



r = Coefficient of reflection for rarer medium

r' = Coefficient of reflection for denser medium

t = Coefficient of refraction from rarer to denser

(interference in transmitted system)

$$= (2e \sec \theta)_1 - (2e \tan \theta \sin i)_2$$

$$= 2e \mu \sec \theta - 2e \frac{\sin^2 \theta}{\cos \theta} \frac{\sin i}{\sin \theta}$$

$$= \frac{2e \mu}{\cos \theta} - 2e \mu \frac{\sin^2 \theta}{\cos \theta}$$

$$= \frac{2\mu e}{\cos \theta} [1 - \sin^2 \theta] = \frac{2\mu e \cos^2 \theta}{\cos \theta} = 2\mu e \cos \theta$$

Path difference between two successive rays of reflection system

$$\Delta x = 2\mu e \cos \theta$$

$$\therefore \text{Phase difference} = \frac{2\pi}{\lambda} \Delta x$$

$$= \frac{2\pi}{\lambda} (2\mu e \cos \theta)$$

There will be additional phase difference of π due to nature of reflection (Stokes's treatment)

$$\Delta \phi = \therefore \text{Net phase difference} = \frac{2\pi}{\lambda} (2\mu e \cos \theta) + \pi$$

Condition for Max.

$$\Delta \phi = 0, 2\pi, 4\pi, 6\pi, \dots$$

$$\frac{2\pi}{\lambda} (2\mu e \cos \theta) + \pi = 0, 2\pi, 4\pi, 6\pi, \dots$$

$$\therefore \frac{2\pi}{\lambda} (2\mu e \cos \theta) = \pi, 3\pi, 5\pi, \dots$$

$$\therefore \text{Path difference } \Delta x = 2\mu e \cos \theta = \lambda/2, 3\lambda/2, 5\lambda/2, \dots$$

$$\begin{aligned} \frac{e}{\mu_2} &= \cos \theta & \frac{0_{1H}}{0_{03}} &= \sin i \\ 0_{102} &= e \sec \theta & 0_{1H} &= 2e \\ \frac{0_{1H}}{e} &= \tan \theta & & \\ 0_{1H} &= e \tan \theta & & \end{aligned}$$

Colour of thin film:- Oil film of water appears coloured in sun light or soap solution.

This phenomena is known as "Colour of thin film". This is due to multiple reflection of wave length (sun light) at the surface of thin liquid film. The condition for maxima and minima in reflected system.

~~Path~~ Path difference $= 2\mu e \cos \theta = \lambda/2, 3\lambda/2, 5\lambda/2 \dots$ max
 $= 0, \lambda, 2\lambda \dots$ min

Since white light consist of seven ~~color~~ wave length. Therefore condition of max and min are satisfied at different places for different value of d . Therefore the fringe are seen coloured. Also e and θ (thickness of liquid film and angle of refraction) are variable ^{at different} ~~at different~~ places, so condition for max and min are satisfied at different places. ~~Due to this reason also liquid film looks coloured in sun light.~~

~~So condition for max and min are satisfied at different places.~~

Newton's ring experiment:- The apparatus consist of a plano-convex lense placed on a simple glass plate with its convex surface in contact with plate. Clearly air film of variable thickness is in closed between plate and lense. At 'O' where plate and lense touch each other, thickness of air film at 'O' is zero.

$$= (2e \sec \theta)_1 - (2e \tan \theta \sin i)_2$$

$$= 2e \mu \sec \theta - 2e \frac{\sin^2 \theta}{\cos \theta} \frac{\sin i}{\sin \theta}$$

$$= \frac{2e \mu}{\cos \theta} - 2e \mu \frac{\sin^2 \theta}{\cos \theta}$$

$$= \frac{2\mu e}{\cos \theta} [1 - \sin^2 \theta] = \frac{2\mu e \cos^2 \theta}{\cos \theta} = 2\mu e \cos \theta$$

$$\begin{aligned} \frac{e}{\mu_2} &= \cos \theta & \frac{\mu_1}{\mu_2} &= \sin i \\ \mu_1 \mu_2 &= e \sec \theta & \mu_1 \mu_2 &= 2e \tan \theta \\ \frac{\mu_1 \mu_2}{e} &= \tan \theta & & \\ \mu_1 \mu_2 &= e \tan \theta & & \end{aligned}$$

Path difference between two successive rays of reflection system

$$\Delta x = 2\mu e \cos \theta$$

$$\therefore \text{Phase difference} = \frac{2\pi}{\lambda} \times \Delta x$$

$$= \frac{2\pi}{\lambda} (2\mu e \cos \theta)$$

There will be additional phase difference of π due to nature of reflection (Stokes's treatment)

$$\Delta \phi = \therefore \text{Net phase difference} = \frac{2\pi}{\lambda} (2\mu e \cos \theta) + \pi$$

Condition for Max.

$$\Delta \phi = 0, 2\pi, 4\pi, 6\pi, \dots$$

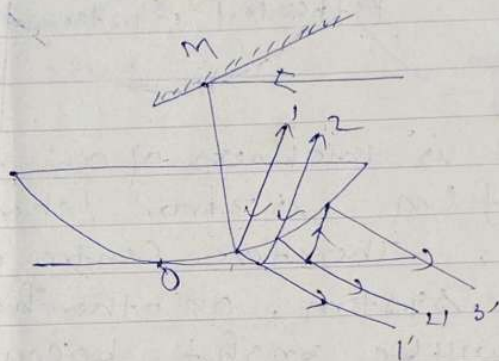
$$\frac{2\pi}{\lambda} (2\mu e \cos \theta) + \pi = 0, 2\pi, 4\pi, 6\pi, \dots$$

$$\therefore \frac{2\pi}{\lambda} (2\mu e \cos \theta) = \pi, 3\pi, 5\pi, \dots$$

$$\therefore \text{Path difference } \Delta x = 2\mu e \cos \theta = \lambda/2, 3\lambda/2, 5\lambda/2, \dots$$

As we moved away from O, thickness of air film goes on increasing. Parallel of monochromatic light are incident normally on this air film with the help of plano-mirror placed at angle 45° with vertical

The ray suffers multiple reflection and refraction at both surface of air film. The ray 1, 2, ... meet to form bright and dark fringe known as interference of reflected system.



Similarly ray $1'2'3'...$ meet to form interference fringes are called transmitted system. This bright and dark fringe can be observed with Ramsden eye piece. Generally fringe in reflected system are observed with microscope.

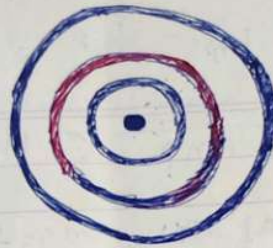
Condition for maxima and minima

Path difference

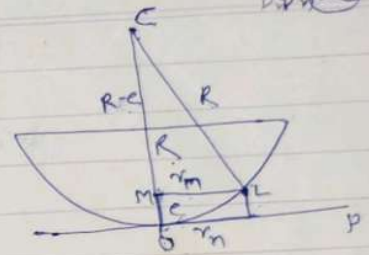
$$2\mu e \cos \theta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \dots \dots \text{max}$$

$$2\mu e \cos \theta = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda \dots \dots \text{min.}$$

for air film $\mu = 1$



$$D_n = \sqrt{2R\lambda n} \quad \text{or} \quad r_n = \sqrt{2R\lambda n}$$



Let thickness of air film at D_n be 'e'
 (D_n = nth dark fringe). Then
 $D_n = r_n$ (radius of nth dark fringe)
 $MC = R - e$

In right angle $\triangle OML$

$$R^2 = (R - e)^2 + r_n^2$$

$$R^2 = R^2 + e^2 - 2Re + r_n^2$$

$$r_n^2 = 2Re$$

$$\therefore 2e = \frac{r_n^2}{R}$$

$$2e = \frac{r_n^2}{R}$$

Path difference $2e = \frac{r_n^2}{R} = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \dots, \left(\frac{2n-1}{2}\right)\lambda$

$$2e = \frac{r_n^2}{R} = 0, \lambda, 2\lambda, \dots, n\lambda \text{ (min)}$$

Since D_n is nth dark fringe for which path difference

$$2e = \frac{r_n^2}{R} = n\lambda$$

$$\therefore r_n^2 = Rn\lambda$$